AD-A050 994 TEXAS TECH UNIV LUBBOCK DEPT OF ELECTRICAL ENGINEERING F/G 9/3
THREE GRAPHICAL TESTS FOR THE STABILITY OF MULTIDIMENSIONAL DIG--ETC(U)
1977 R DECARLO, R SAEKS, J MURRAY AFOSR-74-2631
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19 **REPORT DOCUMENTATION PAGE**

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 18 AFOSR-TR-78-0232
2. GOVT ACCESSION NO.: 2
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): 6 Three Graphical Tests for the Stability of Multidimensional Digital Filters,
5. TYPE OF REPORT & PERIOD COVERED: 9 Interim rept.
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): 10 R./DeCarlo, R./Saeks [redacted] J./Murray
8. CONTRACT OR GRANT NUMBER(s): 15 AFOSR-74-2631
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas Tech University, Dept. of EE, Lubbock, Texas 79409
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 16 2304/A6 17 A6
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, DC 20332
12. REPORT DATE: 11 1977
13. NUMBER OF PAGES: 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 7p.
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release - Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DDC
MAR 8 1978
F

18. SUPPLEMENTARY NOTES

IEEE Intern Symp Circuits Sys p665-669 1977

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nyquist Criterion, Multidimensional Digital Filters, Stability, Homotopy Theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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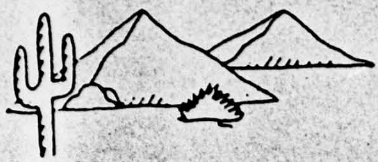
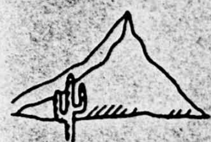
406 820

Part I—All Papers Except Sessions S1 & S3





1977 IEEE INTERNATIONAL SYMPOSIUM ON CIRCUITS AND SYSTEMS

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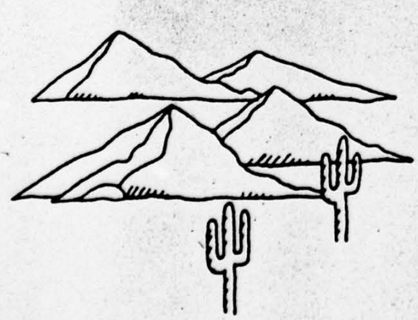

THREE GRAPHICAL TESTS FOR THE STABILITY OF MULTIDIMENSIONAL DIGITAL FILTERS$

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Abstract

This paper discusses three graphical tests for determining the stability of multidimensional digital filters characterized by an appropriate transfer function in several complex variables. Each test is carried out as a finite number of "Nyquist" plots in the complex plane.

Introduction

Recently two of the authors constructed an algebraic topological proof of the Nyquist Criterion (2) (3). The value of this rather sophisticated approach has been harvested in generalizations to systems characterized by transfer functions in several complex variables (1) (2) (3), in particular multidimensional digital filters. Specifically the paper illustrates three graphical tests, similar to the classical Nyquist test, carried out in the complex plane, which determine the stability of a multidimensional digital filter with a transfer function $H(z_1,\dots,z_n) = B(z_1,\dots,z_n)/A(z_1,\dots,z_n)$ where z_i are complex variables and A and B are relatively prime polynomials. The purpose of the paper is to consider these three tests as applied to two different examples.

Background and Main Theorems

Basic to the theory is the 2n (real) dimensional polydisc (8) which is the $\mathbb{C}^n$ analog of the unit disc of $\mathbb{C}$. Mathematically the polydisc P^n is

$$P^n = \{(z_1,\dots,z_n) \text{ in } \mathbb{C}^n \,\big|\, |z_i| \le 1,\ i=1,\dots,n\}$$

There are four separate notions of boundary of the polydisc (1)(8). First is the usual topological boundary

$$B^n = \{(z_1,\dots,z_n) \text{ in } \mathbb{C}^n \,\big|\, |z_i| \le 1,\ i=1,\dots,n \text{ and } |z_k| = 1 \text{ for some } k\}$$

Second is the distinguished boundary

$$T^n = \{(z_1,\dots,z_n) \text{ in } \mathbb{C}^n \,\big|\, |z_i| = 1,\ i=1,\dots n\}$$

T^n serves as the multidimensional analog of the unit circle. In particular the frequency response (7) of a digital filter is the evaluation of its transfer function over T^n.

Thirdly we have

$$M^n = \{(z_1,\dots,z_n) \text{ in } \mathbb{C}^n \,\big|\, |z_i| = 1,\dots,k-1,k+1,\dots,n;\ |z_k| \le 1\}$$

where k ranges from 1 through n. This is a boundary notion in the sense that n-1 coordinates take on extremal values. Finally, the last notion of "boundary" is

$$H^n = \{(z_1,\dots,z_n) \text{ is } \mathbb{C}^n \,\big|\, |z_i| = 1,\ i=1,\dots,k-1;\ |z_k| \le 1;\ z_i = 0,\ i=k+1,\dots,n\}$$

where again k varies from 1 to n. The importance of this concept was first noted by Huang (5). Later it was generalized in (7).

With these notions of boundary one may prove the following Theorem. The proof, however, of the following equivalence is found in the references (1)(2)(5)(7)(9).

<u>Theorem 1</u>: Let a causal multidemsional digital filter be characterized by a rational transfer function in several complex variables. Assume the numerator and denominator polynomials are relatively prime. Then the following are equivalent stability conditions:

(i) the pole set of the transfer function has a null intersection with P^n

(ii) the pole set of the transfer function has a null intersection with B^n

(iii) the pole set of the transfer function has a null intersection with H^n

(iv) the pole set of the transfer function has a null intersection with M^n.

The trouble with these conditions is that the actual test is carried out in a higher dimensional space. For example P^n is 2n dimensional, B^n is (2n-1) dimensional, while H^n and M^n are both (n+1) dimensional. Intuitively, the equivalences of this theorem follow because the pole set of a rational function in several complex variables is an infinite continuum which must intersect the different boundaries of the polydisc if it intersects the polydisc at all. With the intuition gained in (2)(3)(7) the

$ Supported in part by AFOSR Grant 74-2631D

authors were able to simplify these results to graphical tests in the complex plane. The following three theorems are the fruit of this endeavor. Before stating these theorems, one final definition is in order. The "Nyquist plot" of a polynomial, f('), in one complex variable is defined to be the image of T^1 under the map f(') where T^1 is the unit circle of the complex plane. With this in hand, we have the following three tests. Again the proofs can be found in the references (1)(2)(3)(9).

Theorem 2: Let A be the denominator polynomial of a multidimensional digital filter as characterized in Theorem 1. A is a polynomial mapping $\mathbb{C}^n$ to $\mathbb{C}$. Then A has no zeros in P^n (i.e. the filter is stable)* if and only if

(i) A has no zeros on T^n, and

(ii) The Nyquist plots for the single variable functions $A(1,\ldots,1,z_k,1,\ldots,1)$ $k = 1,\ldots,n$ do not encircle zero.

Theorem 3: Let A be as above. Then A has no zeros in P^n (i.e. the filter is stable)* if and only if

(i) A has no zeros on T^n, and

(ii) The Nyquist plots for the single variable functions $A(1,\ldots,1,z_k,0,\ldots,0)$ $k = 1,\ldots,n$, do not encircle zero.

Theorem 4: Let A be as above. Then A has no zeros in P^n (i.e. the filter is stable)* if and only if

(i) A has no zeros on T^n, and

(ii) The Nyquist plot for the single variable function $A(z,z,\ldots,z)$ does not encircle zero.

Each of these tests has essentially the same two parts. First one performs the appropriate encirclement test(s); if zero is not encircled, one then proceeds to check the image of the distinguished boundary. This order of testing (encirclement first, then frequency response) seems in most cases to be preferable to the reverse order, since much less computation is involved in the encirclement tests; however, in cases where the frequency response is known a priori, or must be plotted in any case, the order is immaterial.

It might appear that the third test (Theorem 4) is the best, since it involves only one encirclement test; however, in many cases the relative complexity of the polynomial $A(z,z,\ldots,z)$ will more than offset this advantage. Similarly, in many cases, Theorem 3 may be much easier to apply than Theorem 2. (This is illustrated in the first example). Theorems 2 and 4, however, do have two advantages. The first is mainly philosophical; these Theorems give a test for stability purely in terms of the frequency response of the function A, which corresponds closely with the idea of the Nyquist criterion in one variable. The second advantage is that by filling in the interior of the encirclement plot(s) and taking this region together with the image of the distinguished boundary, one obtains the image of the entire polydisc, from which one can get an accurate idea of stability margins. (The point here is that we have found the image of a 2n-dimensional set--the polydisc--by plotting an n-dimensional set and a 1-dimensional set).

*See note 2.

EXAMPLES

In this section we apply each of the above tests to two examples.

Example 1:

$$A(z_1,z_2) = 5/4\, z_1^2 z_2^2 + \tfrac{1}{4}\, z_1 z_2^3 + \tfrac{1}{4}\, z_1^3 z_2 + 3\, z_1 z_2^2 + 3\, z_1^2 z_2 - z_1^2 - z_2^2 + 3\, z_1 z_2 - 2\, z_1 - 2\, z_2 + 1$$

In this case, we have

$$A(z,1) = 1/2\, z^3 + 13/4\, z^2 + 9/2\, z - 2$$

$$A(1,z) = 1/2\, z^3 + 13/4\, z^2 + 9/2\, z - 2.$$

These polynomials are identical; the image of the unit circle being given in Fig. 1(a).* Since this curve encircles the origin, we deduce immediately that the filter is unstable; for purposes of illustration, we will carry out the other tests.

$$A(z,0) = -z^2 - 2z + 1$$

A(1,z) is as before (Fig. 1(a)); A(z,0) is plotted (for $z = e^{i\theta}$) in Fig. 1(b). Again either plot suffices to verify instability, and clearly A(z,0) gives the simpler test. To apply the third test, we calculate

$$A(z,z) = 9/4\, z^4 + 6\, z^3 + z^2 - 4z + 1$$

and the image of the unit circle under this mapping is plotted in Fig. 1(c). The relative complexity is apparent; however, it again verifies instability. Finally, we plot the image of the distinguished boundary in Fig. 1(d); it can be seen that it does not include the origin, although it does in some sense "encircle" it.

The second example shows that this last kind of "encirclement" is irrelevant; nothing can be deduced from it.

Example 2:

$$A(z_1,z_2) = (z_1 + 2)^3\,(z_2 + 2)^3$$

As before, the plots for A(z,1) and A(1,z) are identical

$$A(z,1) = 27(z + 2)^3$$
$$A(1,z) = 27(z + 2)^3;$$

This plot is given in Fig. 2(a); it does not encircle 0.

*(This illustrates the obvious fact that if the polynomial is symmetric in $z_1,\ldots,z_n$, then the n plots in Theorem 2 in fact reduce to 1 plot--usually simpler than the plot in Theorem 4. Such symmetry is quite common).

In this case, the plot for $A(z,0)=8(z+2)^3$ differs from the previous plot only by a scale factor; we do not draw it separately. Finally, the plot for $A(z,z)=(z + 2)^6$ is given in Fig. 2(b); again it does not encircle the origin.

Thus, in order to determine stability in this case, it is necessary to plot the image of the distinguished boundary; this is done in Fig. 2(c). Since this image does not contain the origin (although it does surround it), we conclude that the filter is stable. This of course is obvious analytically; the present example is merely to illustrate the tests.

Note 1: Because of the magnitudes of the numbers involved, Fig. 2(a) - 2(c) are not drawn to scale.

Note 2: Nonvanishing of the denominator polynomial implies stability; the converse, however, is not quite true unless we assume that the transfer function has no indeterminacies on the distinguished boundary [see(10)].

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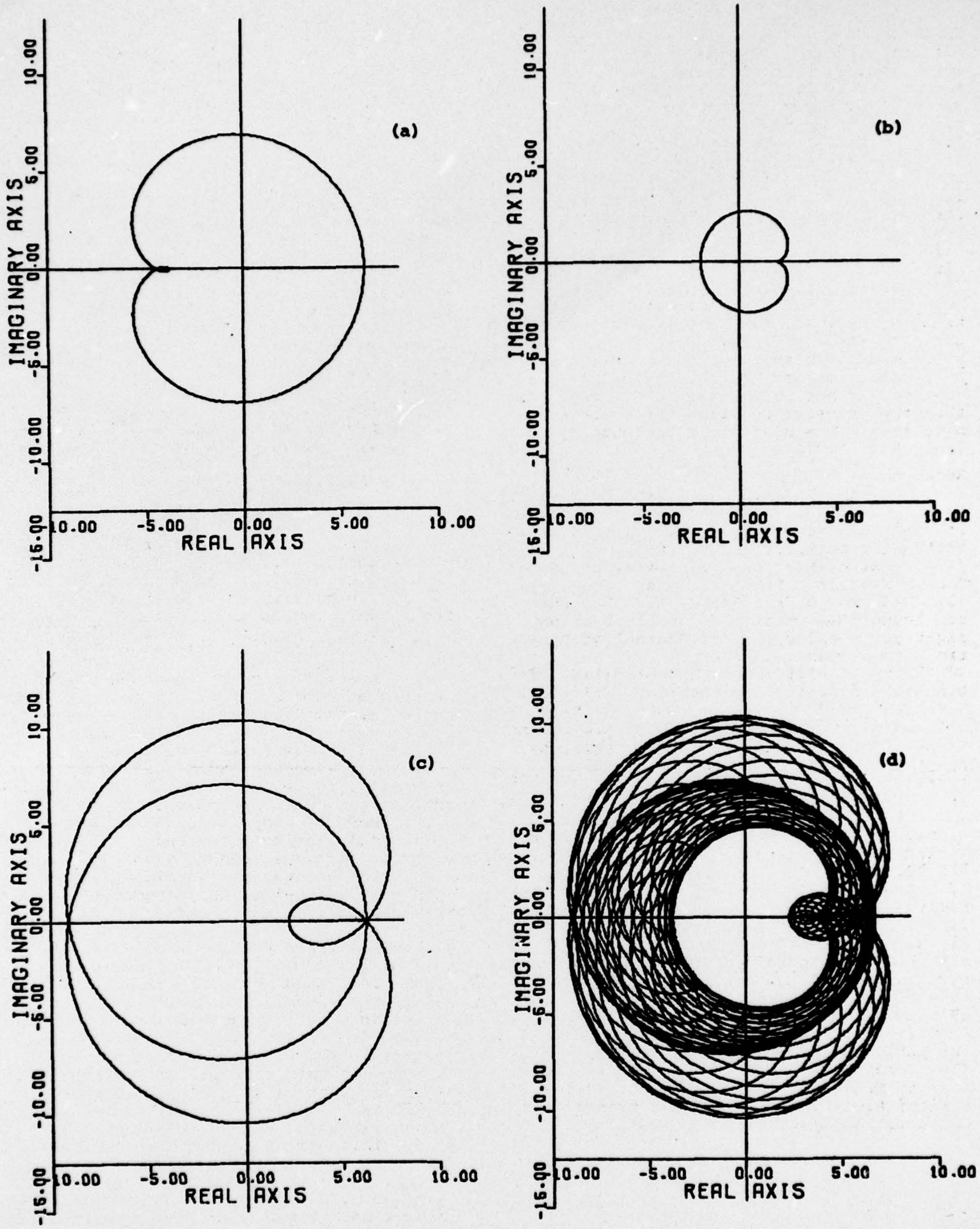
(a)
(b)
(c)
(d)
IMAGINARY AXIS
REAL AXIS
10.00
5.00
0.00
-5.00
-10.00
-15.00
-10.00
-5.00
0.00
5.00
10.00

Fig. 1

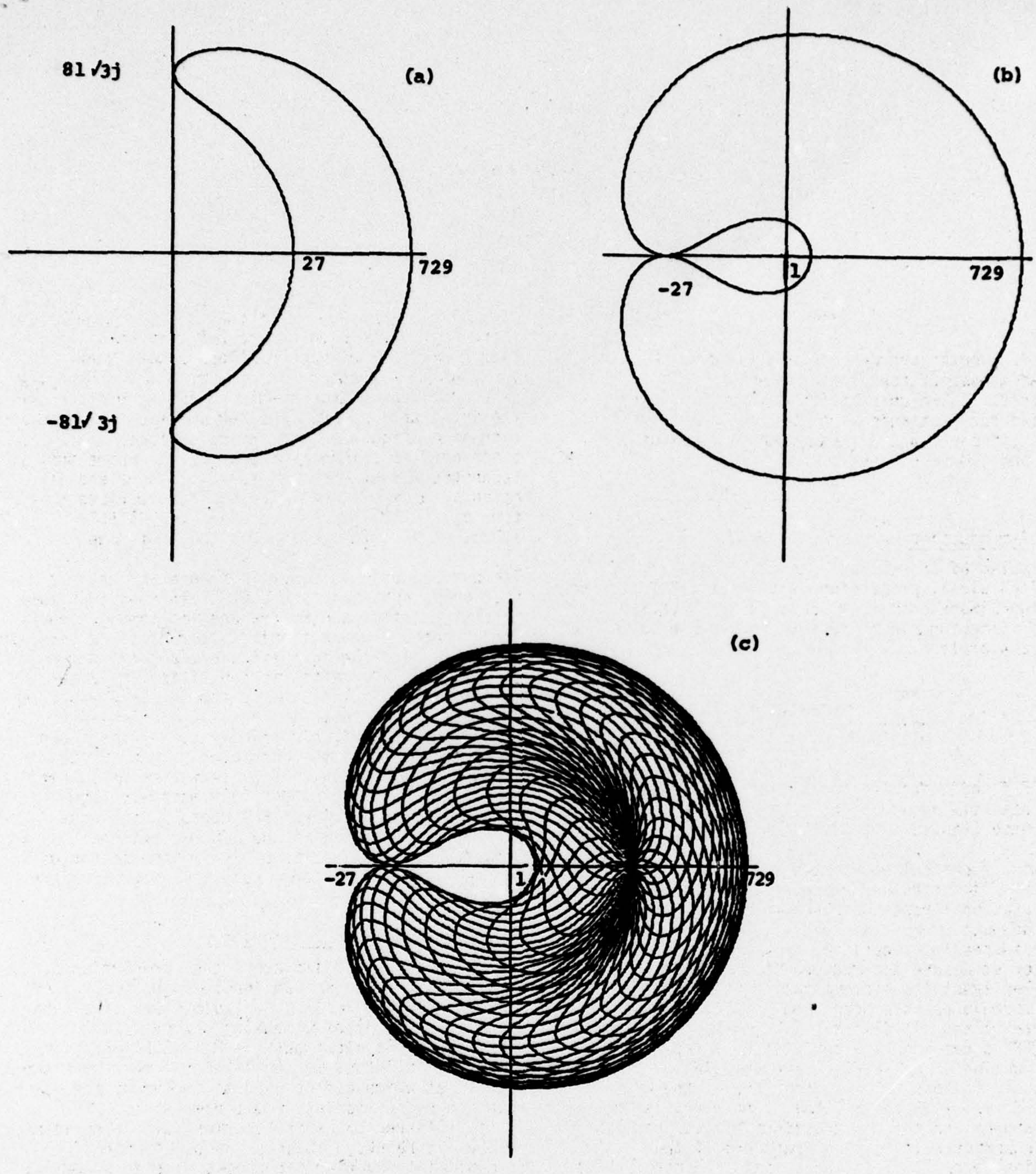

Fig. 2

Note: These plots have been distorted by the mapping

$$r \exp(j\theta) \rightarrow (\ln(1+r)) \exp(j\theta)$$